ELEN 4810 Final Exam

Monday, December 16, 2024, 4:10-7:00 PM. Two sheets of handwritten notes are allowed. No electronics of any kind are allowed. Please record your answers in the exam booklet. Raise your hand if you need additional scratch paper.

There are a total of 4 questions. Good luck!

Name:

Uni:

1. $\mathcal{Z}$-transform. Consider a rational transfer function

$$H(z) = \frac{z^{-2}}{(1 - \frac{3}{2}z^{-1})(1 - j\frac{1}{2}z^{-1})(1 + j\frac{1}{2}z^{-1})} \tag{1}$$

(a) What are the poles and zeros of H?

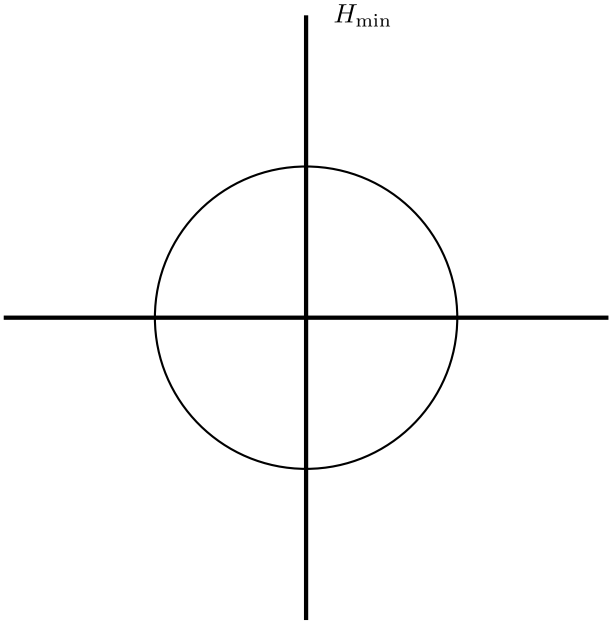
(b) Suppose the system is stable. What is the region of convergence (ROC)?

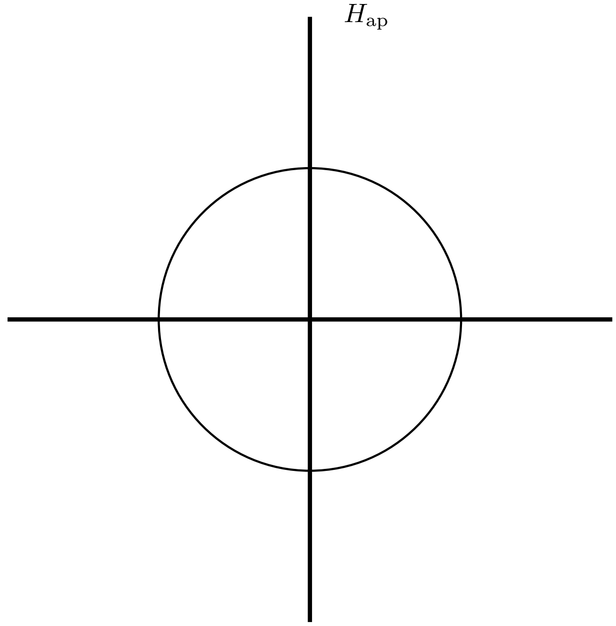
(c) Now suppose the system is causal. What is the region of convergence (ROC)?

(d) Suppose we express $H(z)$ as $H(z) = H_{\min}(z)H_{\mathrm{ap}}(z)$, with $H_{\min}(z)$ minimum phase, and $H_{\mathrm{ap}}(z)$ all-pass. Please sketch the pole-zero diagrams of $H_{\min}$ and H_{ap}, indicating the multiplicity of any repeated poles or zeros.

Answer to Problem 1:

Axes for part (d):

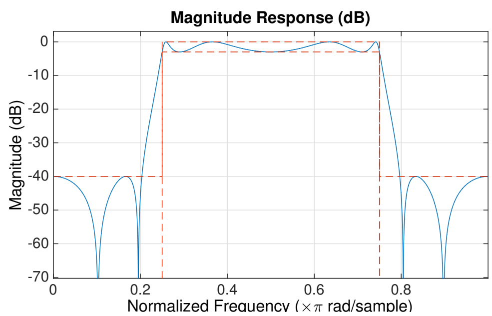




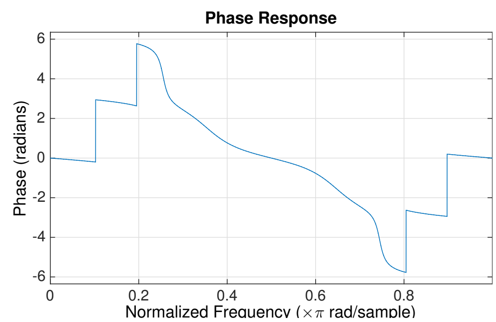
2. Bandpass Filters via Three Approaches. In this problem, we set out with the goal of designing a *bandpass filter* whose magnitude response approximates a target magnitude response

$$|H_{\text{Target}}(e^{j\omega})| = \begin{cases} 1 & \frac{\pi}{4} \leq |\omega| \leq \frac{3\pi}{4} \\ 0 & \text{else,} \end{cases} \tag{2}$$

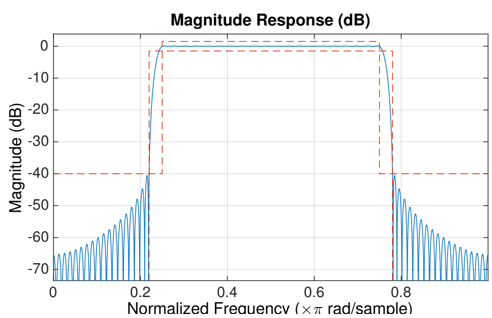
over $-\pi \leq \omega \leq \pi$. We design three different filters, with frequency responses $H_1(e^{j\omega})$, $H_2(e^{j\omega})$, $H_3(e^{j\omega})$. The magnitude and phase response of each are shown below.



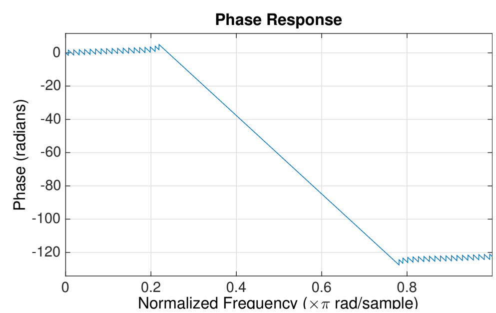
$20\log_{10}|H_1(e^{j\omega})|$



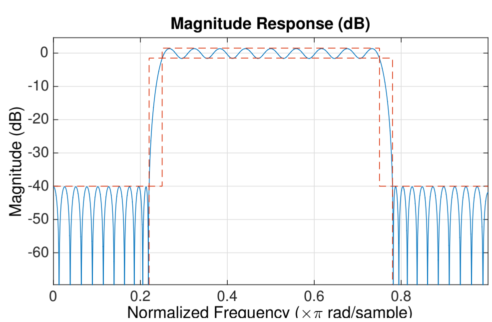
$\angle H_1(e^{j\omega})$



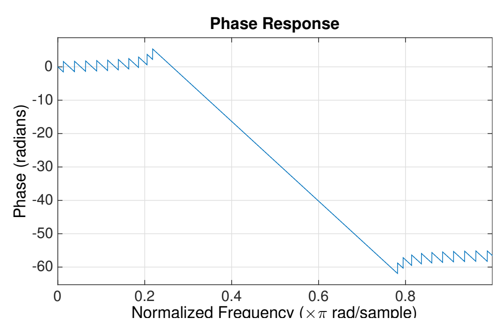
$20\log_{10}|H_2(e^{j\omega})|$



$\angle H_2(e^{j\omega})$



$20\log_{10}|H_3(e^{j\omega})|$



$\angle H_3(e^{j\omega})$

PART 1. How were these filters designed? Consider the following filter design methodologies:

(A) Window. Windowing a target impulse response $h_{\text{Target}}[n]$ with a Kaiser window, to produce a Generalized Linear Phase FIR filter.

(B) Parks-McClellan (L^∞ optimization). Designing an optimal Type I Generalized Linear Phase FIR filter via the Parks-McClellan algorithm.

(C) Minimum Energy. Designing a length-N FIR filter, by choosing the nonzero portion of the impulse response, $h[0], \ldots, h[N-1]$, to minimize the squared error

$$\int_{\omega=-\pi}^{\pi} \left| H(e^{j\omega}) - H_{\text{target}}(e^{j\omega}) \right|^2 d\omega. \tag{3}$$

(D) IIR Elliptic. Transforming a continuous-time *elliptic filter* $H_a(s)$ to discrete time, by setting $H(z) = H_a(\varphi(z))$, where $\varphi(z)$ is some rational function of z.

(E) IIR Butterworth. Transforming a continuous-time *Butterworth filter* $H_a(s)$ to discrete time, by setting $H(z) = H_a(\varphi(z))$, where $\varphi(z)$ is some rational function of z.

For each of the three designs H_1, H_2, H_3, please indicate which design methodology was used. For full credit, please explain how you arrived at your answers.

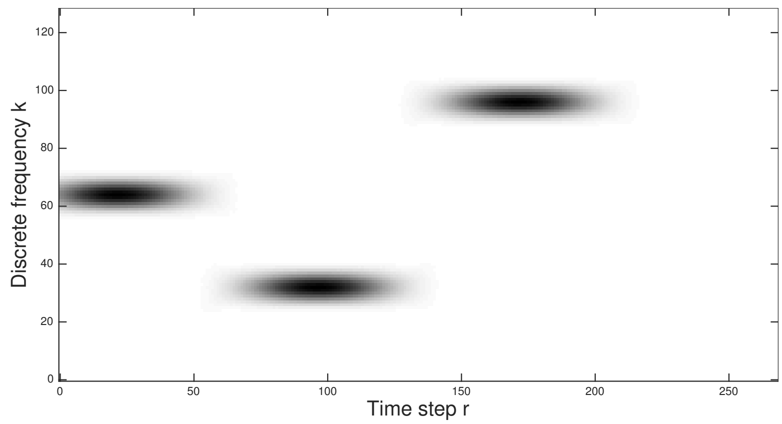
PART 2. Advantages and disadvantages. Why might we prefer System 2 (with $H_2(e^{j\omega})$) to System 1 ($H_1(e^{j\omega})$)? Why might we prefer System 1 ($H_1(e^{j\omega})$) to System 3 ($H_3(e^{j\omega})$)?

Answer to Problem 2:

3. Linear Systems and STFT. Recall the definition of the Short-Time Fourier Transform

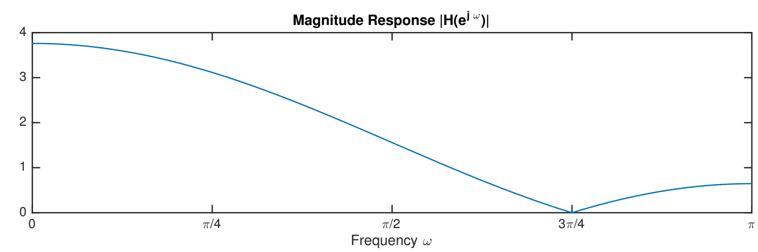
$$X[r,k] = \mathrm{DFT}_N \left\{ w[n]x[n+rR] \right\} [k]$$

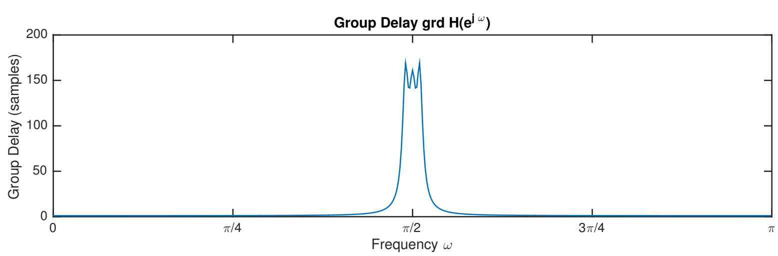
We compute $X[r,k]$ for a signal $x[n]$, with the following parameters: $N = 256$, $R = 2$, and $w[n]$ a Hamming window of length 64. Below, we show the magnitude $|X[r,k]|$ for $r = 0, \ldots, 268$ and $k = 0, \ldots 128$.



In the above figure, the three largest values of $|X[r,k]|$ occur at (i) $r = 21$, $k = 64$, (ii) $r = 96$, $k = 32$, and (iii) $r = 171$, $k = 96$. At each of these peaks, $|X[r,k]| = 1$.

The signal $x[n]$ is passed through a linear, time-invariant system with frequency response $H(e^{j\omega})$ to produce an output $y[n]$. The magnitude response $|H(e^{j\omega})|$ and group delay $\mathrm{grd}[H(e^{j\omega})]$ are shown below:

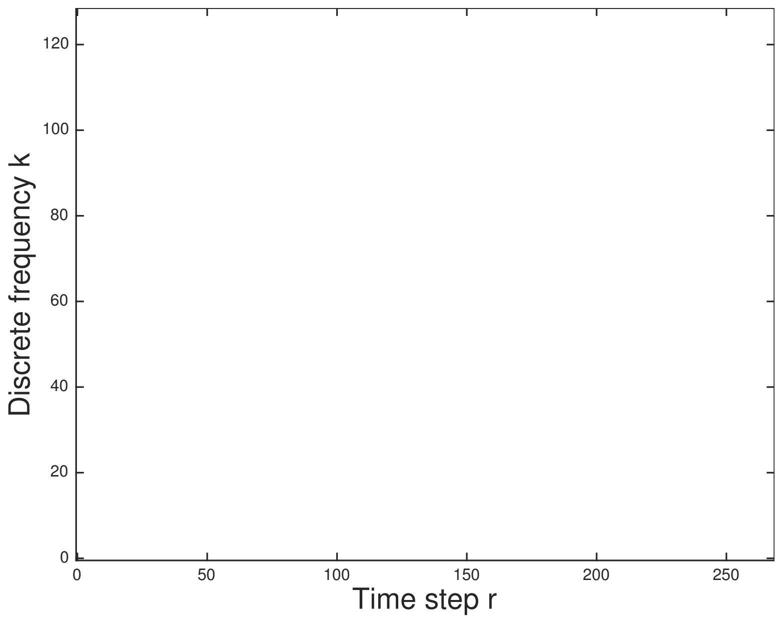




We compute the Short-Time Fourier Transform $Y[r,k]$, with the same parameters. On the axis below, please sketch the spectrogram $|Y[r,k]|$. Please label the approximate location $(r,\ k)$, and height $|Y[r,k]|$ of any peaks in the spectrogram.

[Please briefly explain the reasoning behind your sketch.]

Answer to Problem 3. Please sketch your answer below:



4. Bilinear Transform. A continuous-time system with transfer function $H_c(s)$ has poles at $s = -\frac{1}{3}, -3$ and zeros at $s = \frac{1}{3}, 3$. We design a causal, stable discrete time system $H(z)$ by applying the bilinear transformation to $H_c(s)$, setting

$$H(z) = H_c\left(\frac{z-1}{z+1}\right)$$

Part (i). Please sketch the poles, zeros and region of convergence of H(z)

Part (ii). Which of the following best describes $H(z)$?

LOWPASS BANDPASS HIGHPASS ALLPASS BANDSTOP MINIMUM PHASE

Part (iii). Now, consider a modified system with transfer function

$$H'(z) = H(z)\left(\frac{1+\frac{1}{4}z^{-2}}{1-\frac{1}{4}z^{-2}}\right).$$

At what frequency or frequencies ω is the magnitude response $|H'(e^{j\omega})|$ *maximized*?

Answer to Problem 4: